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FINNISH ELEMENTARY TEACHERS' ESPOUSED BELIEFS ON MATHEMATICAL PROBLEM SOLVING

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According to the Finnish curriculum (NBE 2004), improving pupils' problem solving skills is an important objective in teaching mathematics. Teachers' role is crucial in carrying out the objectives of the curriculum. Especially their conceptions influence decisions they make when teaching mathematics. The purpose of this paper is to find out what kind of conceptions elementary teachers (teachers for grade 3) have concerning problem solving and its teaching in mathematics, as well as what is their understanding about problem solving and how they implement it. The data was gathered with a questionnaire consisting of open questions during November 2010. The questionnaire was sent to a random sample (N = 100) of all Finnish grade 3 teachers. Two rounds of posting produced 63 answers. According to teachers' responses, problem solving in mathematics means in the first place word problems, and problem solving is mainly done collaboratively.

In Finland we have a nine-year comprehensive school where all children learn in heterogeneous classes, also in mathematics. The class size varies between 20–30 pupils, and therefore, teachers have difficulties in balancing between low-achievers and successful pupils, especially in upper grades (grades 7–9). See more on mathematics and teacher education in Finland in Pehkonen, Ahtee & Lavonen (2007).

THEORETICAL FRAMEWORK

Problem solving has been emphasized in the mathematics curriculum of presented in the national core curriculum for basic education (NBE 2004). In the core curriculum, it is defined that mathematics teaching in the comprehensive school delivers to pupils mastering of mathematical concepts and the most common solution methods in basic mathematics. Another objective of teaching is that it should lead pupils to find, elaborate and solve problems. Mathematics teaching influences pupils' spiritual growth and teaches purposeful performance. In the descriptions of key contents and good achievement for different grades, it is repeated the emphases of mathematical problem solving and thinking skills, from even the very first years of school (ibid).

Problem solving in mathematics teaching

Problem solving has generally been accepted as a mean for advancing thinking skills (e.g. Schoenfeld 1985). But the basic concepts, 'problem' and 'problem solving' seem still to be rather ambiguous in the mathematics education. Sometimes a 'problem' is understood to be a simple arithmetic task that can be solved in a routine way, whereas at other times it means a more complex situation. The fuzziness of problem solving concepts is discussed e.g. in Pehkonen (2001).

We shall adopt here the following interpretation that is widely used in the literature for the concept of problem (e.g. Kantowski 1980): A task is said to be a *problem* if its solution requires that an individual combines previously known data in a new way (new at least for the solver).

The nature of problem solving has been described in the literature with the help of problem solving models (e.g. Polya 1945, le Blanc & al. 1980, Mason & al. 1985, Schoenfeld 1985, Aebli 1985). Polya's 60-year-old four-step model is still the most common one: Understanding the problem, Devising a plan, Carrying out the plan, and Looking back (Polya 1945). The shortage of Polya's model has been its oversimplified structure, it looks like a receipt. Mathematical problem solving (e.g. doing new mathematics) is not possible via following such a scheme, instead the solver needs to use their creativity.

Therefore, Polya's model has been modified by other researchers, usually by refining one or more steps. One of these modifications is the model that Schoenfeld (1985) has used. Another one is offered by Mason, Burton and Stacey (1985). Their model has only three phases: Entry, Attack, Review, since they have combined the two middle steps in Polya's model. But their key idea is that between the phases Entry and Attack there is a mulling circle. The mulling in this circle will end until when the solver finds a correct way out, i.e. when one solves the problem (ibid, 131). Similar ideas had earlier been published by Kiesswetter (1983).

Mason and colleagues' interpretation of problem solving is compatible to constructivist understanding of learning (e.g. Davis, Maher & Noddings 1990). Aebli emphasizes the importance of creativity in forming a new concept. That is the difference with Polya who says, that a task is a problem if its solution requires that an individual combines previously known data in a new way (new for the solver). Le Blanc, Proudfit and Putt (1980) identified for elementary school two types of problems: standard textbook problems and process problems.

One may state that thinking of problem solving from a 'models' point of view is not relevant to the study on elementary teachers' problem solving. We disagree this, since according to our observations from elementary classrooms same elements of problem solving can be seen in small pupils' work, although they are not so sophisticated. Similar aspects can be read in published research papers (e.g. Kolovou, van den Heuvel-Panhuizen & Bakker 2009).

On teachers' conceptions

In earlier studies, it has been noticed that teachers' conceptions are of paramount importance when trying to understand teaching situation (e.g. Grouws, Good & Dougherty 1990).

Teaching in school class happens in a large group, but learning is a personal event, and therefore, it affects each individual differently. Ultimately, a teacher's task is to create learning environments that allows high-quality learning for all. Martin Hughes

(1986) have presented a perspective on children's attempts to understand mathematics. It is important that children can create their own strategies for understanding mathematics using problem solving. In order to understand teachers' decisions, it is important to know what a teacher thinks about teaching and ways to help pupils learn, i.e. teachers' conceptions.

Conception is problematic as a concept, since it is not clearly defined (cf. Furinghetti & Pehkonen 2002). Here we understand an individual's beliefs in a rather wide sense as their subjective, experience-based, often implicit knowledge and emotions on some matter or state of art. Furthermore, *conceptions* are explained as conscious beliefs. In the case of conceptions, we understand that the cognitive component of beliefs is stressed, whereas in basic (primitive) beliefs the affective component is emphasized (Pehkonen 1998).

Sivunen & Pehkonen (2009) implemented a study about elementary teachers' conceptions on problem solving and its teaching in mathematics, in the case of one school district (N = 43). According to these teachers, problem solving in mathematics means various problems, strategies, mathematics in everyday situations, pupils' own thinking and applying previously learned skills. Here we want to generalize these results to the whole Finland using a statistically representative sample.

The focus of the paper

This paper examines Finnish elementary teachers' (grade 3) conceptions about mathematical problem solving. Thus the research questions can be formulated as follows: (1) How do teachers understand the concept 'problem'? (2) What do teachers understand as problem solving? (3) What kind of examples do teachers provide for problems? (4) How often do teachers use problems in their classes?

METHOD

The empirical section is a part of a larger research project called "*On the development of pupils' and teachers' mathematical understanding and performance when dealing with open-ended problems*". It is a joint three-year project with Chile (2010–13), financed by the Academy of Finland (project number 135556). Within the project we will try to develop a model for improving the level of understanding, the skills and self-confidence of teachers and pupils, with the active participation of teachers, using certain open-ended problems.

The study dealt with in this paper is a nationwide survey about problem solving. This background study aims to single out what kind of conceptions grade 3 teachers in Finnish elementary schools have concerning problem solving. Then, in the case of random sample, the results can be generalized to all Finnish elementary teachers, since they do not specialized in one grade level, as in some countries, but usually teach all grades in elementary school (grades 1–6).

Indicator

The questionnaire was prepared for this study with the help of some existing papers: Polya (1945), le Blanc & al. (1980), Kantowski (1980) and Aebli (1985). There are three levels of understanding the concept 'problem'; firstly as a word problem, secondly that an individual combines previously known data in a new way (new for the solver), and thirdly formatting a new concept.

The questionnaire was also made up to distinguish how differently teachers used the steps in Polya's model. In addition, we were interested in problem solving in mathematics context and in frame of reference. There were five open questions in the questionnaire. Some alternative answers were offered to the main questions, and an additional option for respondents to write their own answers (cf. Tables 1 and 2). In the beginning of the questionnaire. Our ten experimenting teachers (3 grade) tested the questionnaire, and it was corrected.

The questionnaire in Finnish was sent to a random sample ($N = 100$) of all Finnish comprehensive schools asking their headmasters to give the questionnaire to one grade 3 teacher. If there were many teachers in school, then to the teacher who is first on their list of grade 3 teachers.

Participants

The data was gathered with a questionnaire in autumn 2010. It was selected at random 100 schools from Finland's comprehensive schools. The questionnaire was posted to the headmasters of these schools in the middle of November. A total of 63 responses was received from schools, thus the response rate was good.

Among the 63 respondents, there were 51 female teachers and 12 male teachers, the share corresponding roughly the gender division in teacher education. Since the share of the male teachers was so small, we have kept all teachers as one group. Most of the participants were experienced teachers, the mean value of their school experience was 16.8 years, and it varied from 2 months to 35 years.

Methods of data analysis

Ten first papers were used to create a categorization of answers. After classifying all answers into these categories, they were written into a SPSS file, in order to calculate statistical indicators (modes etc.) and analyze relevant explanatory factors.

PRELIMINARY RESULTS

In the following, we will present the 3rd grade teachers' conceptions of problem solving, and some examples of problems they stated to be using in their lessons.

The question: What is a problem?

In the first question, the teachers were asked "How would you describe the concept of problem", and they were given four alternatives from which they should number the most important option and the second option (Table 1).

Table 1: The teachers' answers to the question "What is a problem?".

<i>Alternatives given in the question</i>	<i>The 1st option</i>	<i>The 2nd option</i>
A problem is a verbal task, where the pupils apply their previously knowledge	27	15
A problem is a verbal task to be solved using reasoning	17	19
A problem is a task the solution of which is not in my mind	15	12
A problem is something I want to find out but where I have gaps in my knowledge	8	12
What else might a problem mean	1	5

Some teachers gave more than two responses as their 1st option, and therefore, the sum of first column is bigger than 63. In the following, the frequencies of choices are given in brackets (the 1st choice; the 2nd choice). The mode is in the first line: The teachers seem to understand problem as a verbal task (27;15) in which pupils apply their earlier knowledge. Many teachers answered that a problem is a verbal task that will be solved by reasoning (17;19). Thus, altogether 44 respondents (70%) attached a verbal task as their first alternative a verbal task to the concept 'problem'.

The remaining 19 respondents (30 %) linked with the concept 'problem' following ideas: For about one fourth of the teachers, problem is a task for which the solution is not known (15;12). Problem was also defined as something you would like to know, but you have gaps in your knowledge base (8;12). Some representative additional statements are, as follows: "Problem should be understood more broadly than only as solving of verbal tasks." In additional answers, one teacher states that "Problem in mathematics is all things above, depending on the class /on pupils' mathematical knowledge / skills and abilities."

Summarizing, the most common understanding among the grade 3 teachers was that problem is a verbal task.

The question: What is problem solving?

In the second question, the teachers were asked “What is problem-solving in mathematics?” Here we developed from the most important options and the second options of the alternatives the following table (Table 2).

Table 2: The teachers’ answers to the question “What is problem solving?”

<i>Alternatives given in the question</i>	<i>The 1st option</i>	<i>The 2nd option</i>
Helping pupils to find out their solutions for the task	33	15
Using previous knowledge to solve a task	20	16
Defining the conditions and choosing solving methods	9	16
Exercising learning procedures and methods	2	8
Solving a verbal task together with pupils	0	6
Evaluating the solution found	0	1
What else could problem solving be	0	2

Some teachers gave more than two answers, and therefore, the sums of both columns are bigger than 63. The frequencies of choices are given in brackets (the 1st choice; the 2nd choice) in the following.

The mode is in the first line: Most of the teachers would like to see themselves as the pupils’ guide in helping them to find solutions to problems (33;15). Many teachers said that problem solving is applying previously learned skills (20;16). Some of the teachers thought that problem solving is a determination of the terms of the task, and a selection on the solution (9;16). Some of the grade 3 teachers said that problem solving is to exercise procedures and methods (2;8). Still other teachers stated that it is solving a verbal task together with the pupils (0;6).

In the teachers’ free answers (“What else...”) one can see many interesting details that indicate the teachers’ multi-faceted understanding on problem solving. For example, many teachers used at least one of the following concepts: *attitude*, *sensibility*, *motivation*, *creative*, *flow*, *discuss* that were not given in the questionnaire.

In practical situations I connect also an attitude. I don’t give answers but the children have to find solutions themselves, in that way the sensibility and the motivation will be preserved in the schoolwork.

It is a creative individual/ group activity.

How to implement problem solving?

The fifth question of the questionnaire “Give an example of the way of teaching you like to use in a mathematics lesson when dealing with problem tasks” inquired on the way how the teachers will implement problem solving in their class.

When the answers were classified according to two school-practice variables or kinds of working in class (individual work – teamwork; teacher-centered – pupil-centered), the following four-field matrix (Figure 1) was found:

	Teamwork	
	22	28
Teacher-centered	Student-centered	
	3	1
	Individual work	

Figure 1: Practices in school classes.

The sum is here not 63, because some teachers did not answer this question. Another researcher classified the responses independently, and the reliability for the classification was found to be 93 %.

Here I give some examples about this four-field matrix. To the teacher-centered teamwork, there are classified answers like this.

Made a task together on the table is often rewarding.

In the student-centered teamwork, there are the answers like this.

Group of 3-4 pupils. They are trying to work together to solve the problem which has been given. The teacher guides the final solution, if the group can not solve it.

The teacher-centered individual work is like this.

I read the problem to the pupils and give the necessary data/numbers. I give each of them a label to solve the problem. At the end we are discussing about the answer.

The pupil-centered individual work is like this.

A week’s puzzle. We are checking the answer at the end of the week and sometimes rewarding.

In the four-field matrix above, we can clearly see the emphasis on teamwork. Thus on one hand, the majority of the teachers (50) considered problem solving to be teamwork. On the other hand, teacher-centered and a pupil-centered working seemed to be equally popular.

The question: What kind of examples teachers give for problems?

We also asked teachers for examples that they like to use in their problem solving lessons. These answers can be classified into five categories; use of tools, mathematics in everyday situations, verbal tasks, use of textbooks, and examples in meta level. In the following the frequency is given in brackets.

A common belief among teachers was the idea of the usefulness of tools (21). As we saw before, 70 % of the grade 3 teachers considered the problem to be a verbal task, but only about half of them offered here a word problem (15). These problems seem to be the same as in the textbooks. Mathematics in everyday situations (12) was one of the reasons for choosing the problem. The idea here was to prepare pupils to use mathematics in everyday life. There were some teachers (8) whose pedagogical reflection was in meta-level. They wanted to help their pupils in thinking. Some teachers were using textbooks and teacher guides (7) when they chose their problems, because most of the problems they highlighted in the questionnaire seemed to be from the textbooks. According Kolovou & al. (2009) elementary textbooks contain only few problems. As teachers, however, use a lot of textbook tasks, then one should consider how the textbooks could be improved.

The question: How often do teachers use problems in their lessons?

In the questionnaire, it was asked how often did the teachers use problems in their lessons. Altogether 79 % of the grade 3 teachers tell that they use problem solving at least once a week, and 38 % of them answered that they use it in all their lessons. Furthermore, it is questionable that all the teachers said that they use problem solving at least occasionally (19 %). Those who said that they use it occasionally, meant that it requires a suitable context, and therefore there is no regularity.

DISCUSSION

The goal for mathematics teaching is that pupils learn to understand also the structures of mathematics, not just mechanical calculations (NBE 2004). Higher-level thinking in learning can be reached in working with such a problem where an individual has to integrate existing data in a new way in order to find a solution.

The survey results indicate that all grade 3 teachers said that they use problem-solving. However, they mean different things by problem solving and problem-solving tasks. A word problem and a problem-solving task are not automatically synonymous. On one hand word problems can be solved fairly routinely, and on the other hand traditional product tasks can be solved according to the problem solving principles. More than the type of task is the question of how to guide pupils.

Additionally with the use of a questionnaire, we can reach only teachers' espoused conceptions. They may act in their teaching practice totally differently. This has been noticed already some twenty years ago, e.g. Ernest (1989) wrote on two kinds of teacher's conceptions, namely 'espoused' and 'enacted'. But also teachers' espoused conceptions reflect on their actual thinking, and therefore, are worthwhile to study.

It seems that in Polya's model the first step (Polya 1945), understanding the problem by defining the conditions and choosing solving methods of a task, was not emphasized in the teachers' answers. This step is important for two reasons in problem solving. In the interpolation problems, the identification of missing elements activates the necessary concepts, and in open problems orientation step, a student might limit the size of the problem. Another reason is related to motivation. When the teacher is preparing a lesson he/she should think about what motivates pupils, and how their interest can be maintained. (cf. Aebli 1985). In a learning situation, teachers regulate with questions the level of knowledge to be learnt. Genuine listening to pupils (cf. Pehkonen & Ahtee 2005) and focusing on pupils' thinking helps the teacher to do such questions to pupils that assist them into the heart of the problem.

In Polya's second and third step (Polya 1945), devising a plan and carrying out the plan, was focus in the teachers' answers. All the grade 3 teachers answered that they use the problem solving at least occasionally. Also the majority of the teachers considered problem solving as teamwork. They stated that they use everyday situations as the problems, and word problems were very popular (70%). When we looked at the tasks they said to have used, we saw that most of them have named routine verbal tasks to be problems. We cannot see in this study, how the teachers implement their lessons, but careful estimates based on our experiences as teacher educators suggest that about half of the teachers mention the use of routine tasks in Kantowski's (1980) meaning.

The study of Sivunen & Pehkonen (2009) dealt only the elementary teachers in one town, i.e. it was a case study, and therefore, its results were not generalizable. Our study was based on a random sample of grade 3 teachers in Finland, and the response rate was rather good (63 %). Thus its results can be generalized to all Finnish elementary teachers, since Finnish elementary teachers do not specialized in one grade level but usually teach all grades after each other in elementary school (grades 1–6).

So we can say that because half of the grade 3 teachers use routine tasks, their pupils' learning stays on the routine level. It is difficult to change wrong learning habits, and therefore, also the elementary teachers should understand what problem and problem solving means.

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